



# Process Standards for Mathematics in Action!

Building Background for  
Educators

# Project Information

This presentation hopes to answer two questions for educators:

- *How do you integrate the Process Standards for Mathematics into instruction of daily lessons in conjunction with the content?*
- *What does this integration look like for teachers and for students?*

# Process Standards for Mathematics

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

1. Make sense of problems and persevere in solving them

6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Reasoning and explaining

Modeling and using tools

Seeing structure and generalizing

# Process Standard#1

Make sense of problems and persevere in solving them

# What is a problem?

- A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific “correct” solution method.

Hiebert et al.,

1997

Make sense of problems and persevere in solving them

Students should be able to...

- Explain the meaning of a problem
- Describe possible approaches to a solution
- Consider similar problems to gain insights
- Use concrete objects or illustrations to think about and solve problems
- Monitor and evaluate their progress and change strategy if needed
- Check their answers using a different method

# Make sense of problems and persevere in solving them

## Grade 5

*There are 228 players in the softball league.  
How many 12-member teams can be  
formed if each player is placed on exactly  
one team?*



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# Make sense of problems and persevere in solving them

## Kindergarten

*I have 5 things on my plate.*

*Some are peas. Some are carrots.*

*How many of each could I have?*

*How many peas? How many carrots?*

- Adapted from *Investigations in Number, Data, and Space*



Make sense of problems and persevere in solving them

## Teachers should...

### ● Provide Good Problems

To determine whether it's a good problem, ask yourself:

1. Is the problem interesting to students?
2. Does the problem involve meaningful mathematics?
3. Does the problem provide an opportunity for students to apply and extend mathematics they are learning or have learned?
4. Is the problem challenging for students?
5. Does the problem support the use of multiple strategies?
6. Will students' interactions with the problem reveal information about students' mathematics understanding?

Make sense of problems and persevere in solving them

## Teachers should...

- Facilitate student engagement in the problem-solving process
  - ✓ Provide access to appropriate tools/resources
  - ✓ Encourage discussion and questions
- Support students to
  - ✓ Make sense of the problem
  - ✓ Make connections
  - ✓ Check reasonableness of solutions

# Process Standard#6

Attend to precision.

Attend to Precision

Students should be able to...

- Communicate precisely to others
- State meaning of symbols and definitions and use them appropriately
- Specify units of measure
- Calculate accurately and efficiently

# Attend to Precision

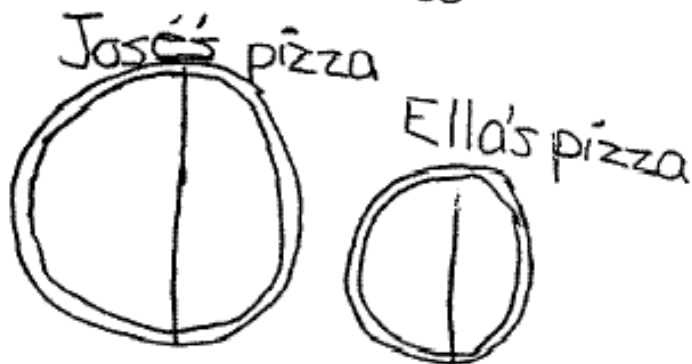
## Grade 3

José ate  $\frac{1}{2}$  of a pizza.

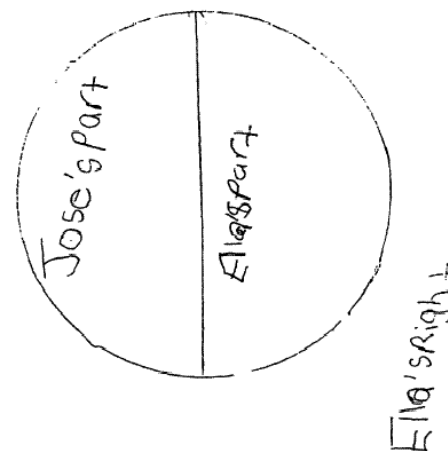
Ella ate  $\frac{1}{2}$  of another pizza.

José said that he ate more pizza than Ella, but Ella said they both ate the same amount. Who is right? Support your answer with words and pictures.

José could be right because his pizza could be bigger than Ella's.



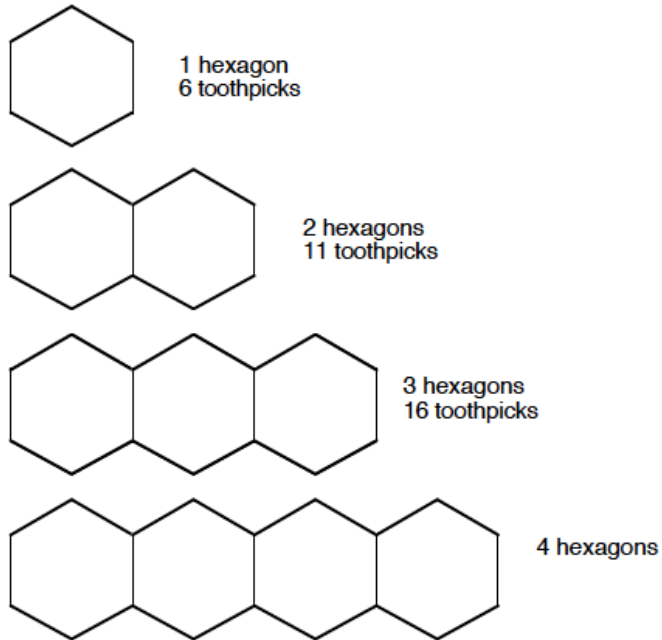
Student A  
Response



Student B  
Response

# Attend to Precision

## Grade 6



How many toothpicks are needed to make 4 hexagons? 5 hexagons? 10 hexagons?  $n$  hexagons?

### Student A Response

*I found a pattern of adding five toothpicks for each hexagon. Since 21 toothpicks are in 4 hexagons, I know that five more makes 26. I could also multiply 5 by 6 and subtract 4.*

### Student B Response

*To find the number of toothpicks in 5 hexagons, add five to 21 (the number in 4 hexagons).*



Attend to Precision

## Teachers Should...

- Model appropriate use of mathematics vocabulary, symbols, and explanations
- Provide opportunities for students to share their thinking
- Prepare students for further study



# Process Standard#2

Reason Abstractly and Quantitatively.

## Reason Abstractly and Quantitatively

# Students should be able to...

- Make sense of quantities and their relationships in problem situations
- Contextualize and decontextualize
- Create a coherent representation of the problem at hand

# Reason Abstractly and Quantitatively

## Grade 1

Decontextualize:

Write a number sentence to represent this story –

There are 12 girls and 13 boys in Mrs. Johnson's class? How many students are in Mrs. Johnson's altogether?

$$12 + 13 = \square$$

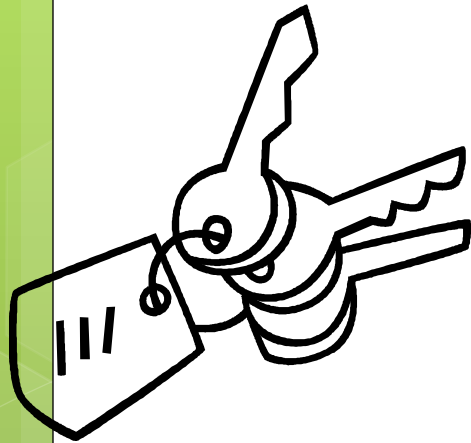
Contextualize:

Write a story that could be represented by the number sentence  $15 - 9$

Liam had 15 pencils. He gave 9 pencils to his classmates. How many pencils does Liam have left?

Reason Abstractly and Quantitatively

Key Words are not the Keys to Mathematical Reasoning

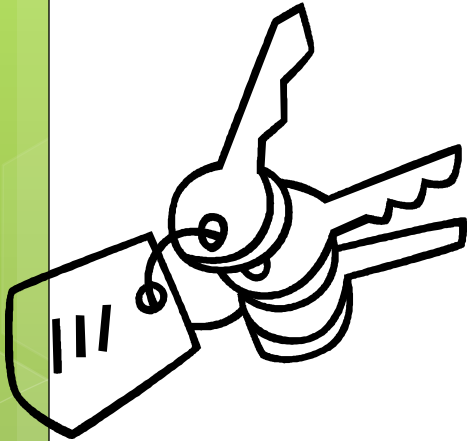


**Katja invites 8 children  
to come to her  
birthday party,  
which takes place  
in 4 days.  
How old will Katja be  
on her birthday?**

*From Making Sense of Word Problems,  
Verschaffel, Greer, DeCorte,  
Swets & Zeilinger B.V.,  
Lisse, The Netherlands, 2000*

Reason Abstractly and Quantitatively

## Key Words are not the Keys to Mathematical Reasoning



Bernard had some coins. Samuel gave him 6 more. Now Bernard has 14 coins. How many coins did Bernard have to begin with?



# Reason Abstractly and Quantitatively

## Grade 6

3 pineapples

1 serving =  $\frac{1}{2}$  pineapple

Given the information in the box above, write a mathematics word problem for which  $3 \div \frac{1}{2}$  would be the method of solution.

# Reason Abstractly and Quantitatively

## Teachers Should...

- Provide opportunities for students to
  - Express interpretations about number
  - Apply relationships between numbers
  - Recognize magnitude of numbers
  - Compute
  - Make decisions involving numbers
  - Solve problems

# Reason Abstractly and Quantitatively Teachers Should...

- Draw students' attention to numbers and their applications
- Encourage discussion that promotes reasoning



# Process Standard #3

Construct viable arguments and critique the reasoning of others.

## Construct Viable Arguments and Critique the Reasoning of Others

### Students should be able to...

- Understand and use stated assumptions, definitions, and previous results
- Analyze situations
- Justify conclusions
- Compare two plausible arguments
- Determine domain to which argument applies

# Construct Viable Arguments and Critique the Reasoning of Others

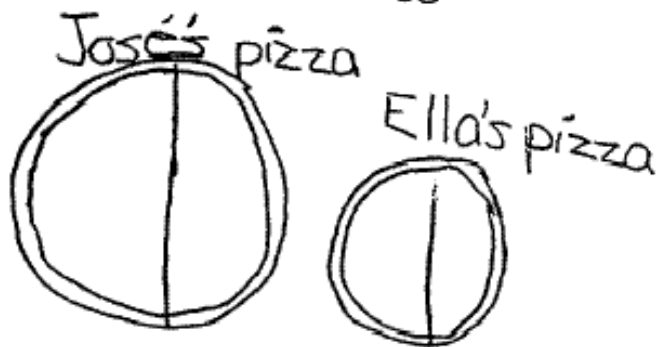
## Grade 3

José ate  $\frac{1}{2}$  of a pizza.

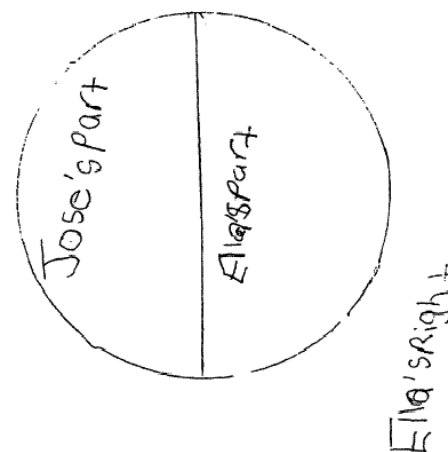
Ella ate  $\frac{1}{2}$  of another pizza.

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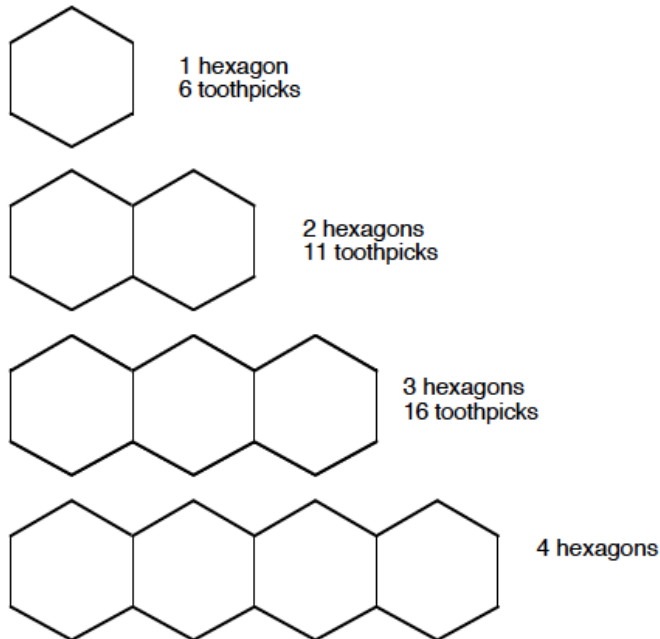
Student A  
Response



Student B  
Response

# Construct Viable Arguments and Critique the Reasoning of Others

## Grade 6



How many toothpicks are needed to make 4 hexagons?

5 hexagons?

10 hexagons?

$n$  hexagons?

### Student A Response

*I found a pattern of adding five toothpicks for each hexagon. Since 21 toothpicks are in 4 hexagons, I know that five more makes 26. I could also multiply 5 by 6 and subtract 4.*

### Student B Response

*To find the number of toothpicks in 5 hexagons, add five to 21 (the number in 4 hexagons).*

Construct Viable Arguments and Critique the Reasoning of Others

## Teachers Should...

- Establish supportive social norms
- Provide opportunities for students to make and evaluate conjectures
- Facilitate meaningful discussions of mathematics

# Process Standard#4

Model with Mathematics.

# Model with Mathematics

Students should be able to...

- Apply mathematics to solve problems in everyday life
- Make assumptions and approximations to simplify a problem
- Identify important quantities and use tools to map their relationships
- Reflect on the reasonableness of their answer based on the context of the problem

# Model with Mathematics

## Grade 1

Real-World Scenario:

David had \$37. His grandpa gave him some money for his birthday. Now he has \$63. How much money did David's grandpa give him?

Symbolic Model:

$$37 + \square = 63$$

Real-World Scenario:

David had some stickers. He gave 37 to Susan. Now he has 26 stickers. How many stickers did David have before?

Symbolic Model:

$$\square - 37 = 26$$



# Model with Mathematics

## Grades 4-5

Whole number division:  $25 \div 4 =$

If 25 pennies are divided equally among 4 kids, how many pennies will each kid receive?

$$25 \div 4 = 6 \text{ R}1$$

If 25 students are going on a field trip, and each vehicle can hold 4 students, how many vehicles are needed?

$$25 \div$$

If Candace shares 25 candy bars equally among 4 people, how many candy bars will each person get?

$$25 \div 4 = 6 \frac{1}{4}$$

If John cuts a 25-meter board into 4 equal pieces, how long will each of the pieces be?

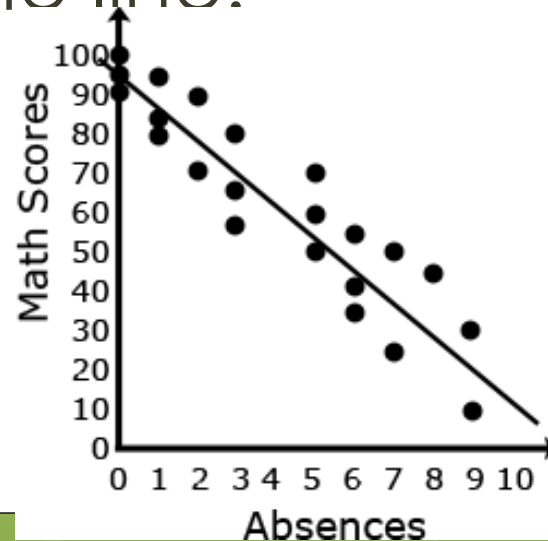
$$25 \div 4 = 6.25$$

# Model with Mathematics

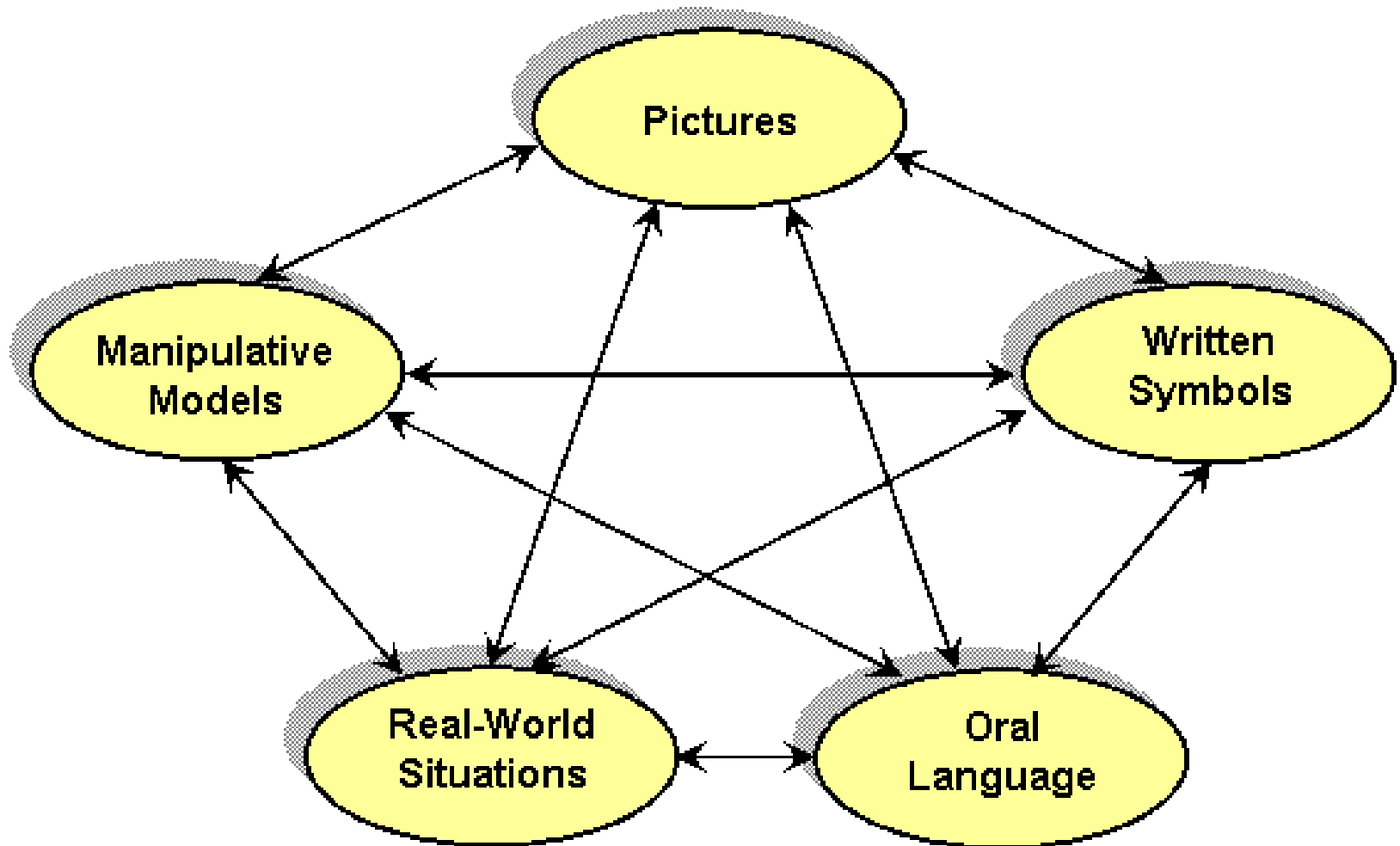
## Grade 8

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80

Given data from students' math scores and absences, make a scatterplot. Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.



# Modes of Representation



# Model with Mathematics Teachers Should...

- Provide opportunities for students to solve real-world problems
- Focus students' attention on sense-making and reasonableness of results
- Have students develop real-life contexts to support mathematical expressions

# Process Standard #5

Use Appropriate Tools Strategically

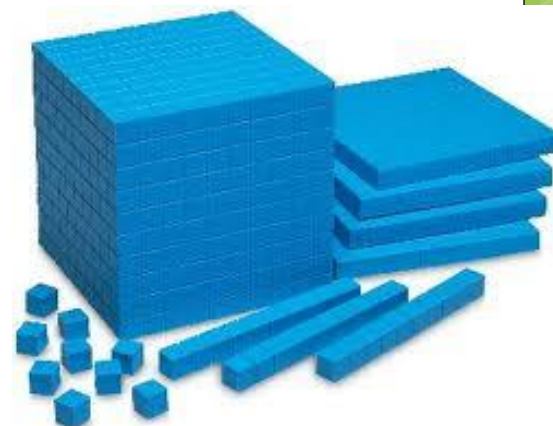
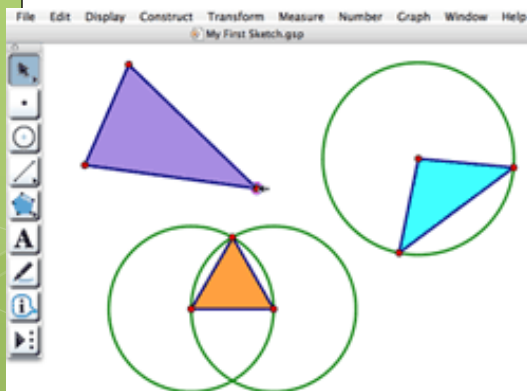
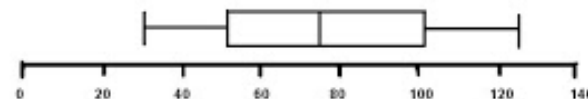




$$D = rt$$

distance      rate      time

What is  
a tool?



# Use appropriate tools strategically

Students should be able to...

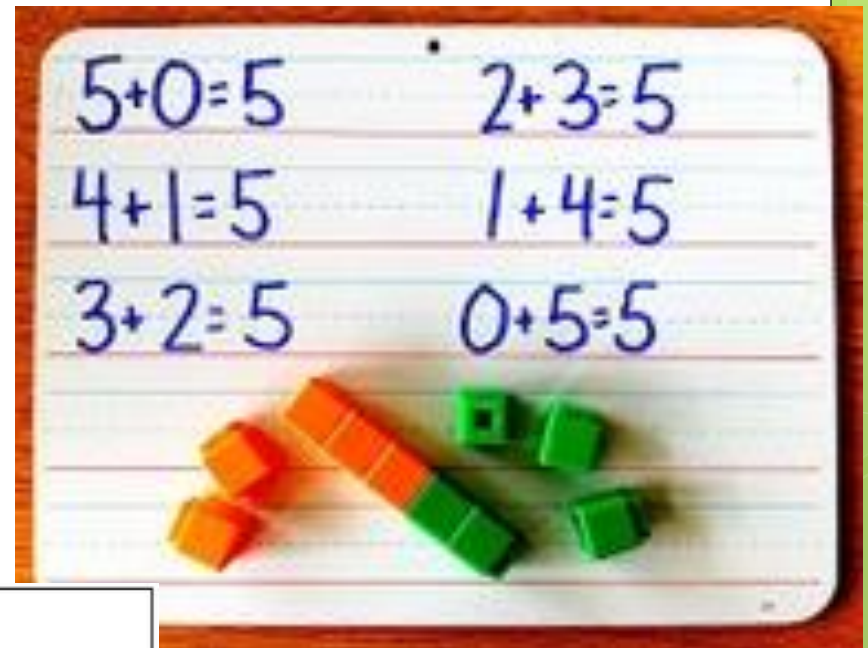
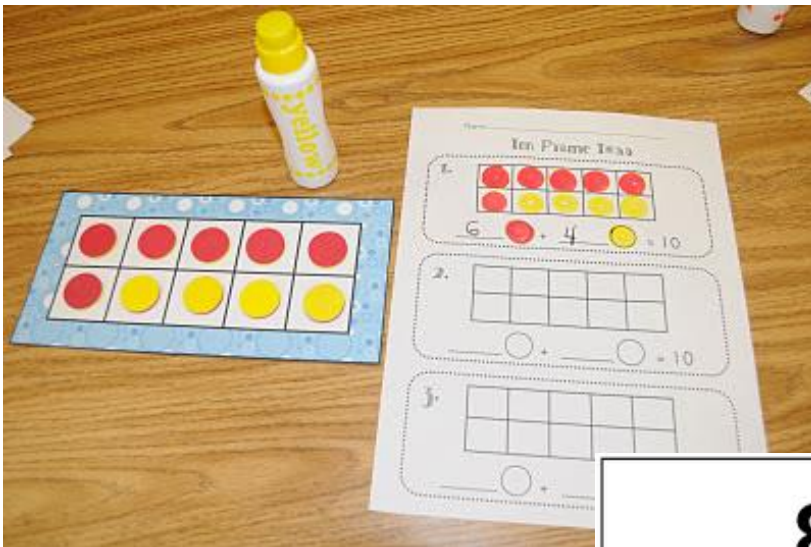
- Consider a variety of tools and choose the appropriate tool to support their problem solving
- Use estimation to detect possible errors
- Use technology to help visualize, explore, and compare information



# Use appropriate tools strategically

## Kindergarten

### Decomposing Numbers



8	
5	3

# Use appropriate tools strategically

## Grade 4

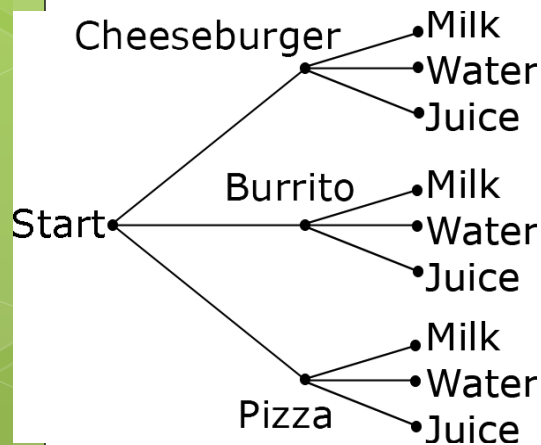
Determine how many unique pairs of Main Dish and Drinks can be made.

### Main Dish

Cheeseburger  
Burrito  
Pizza

### Drink

Milk  
Water  
Juice



	Milk	Water	Juice
Cheeseburger	x	x	x
Burrito	x	x	x
Pizza	x	x	x

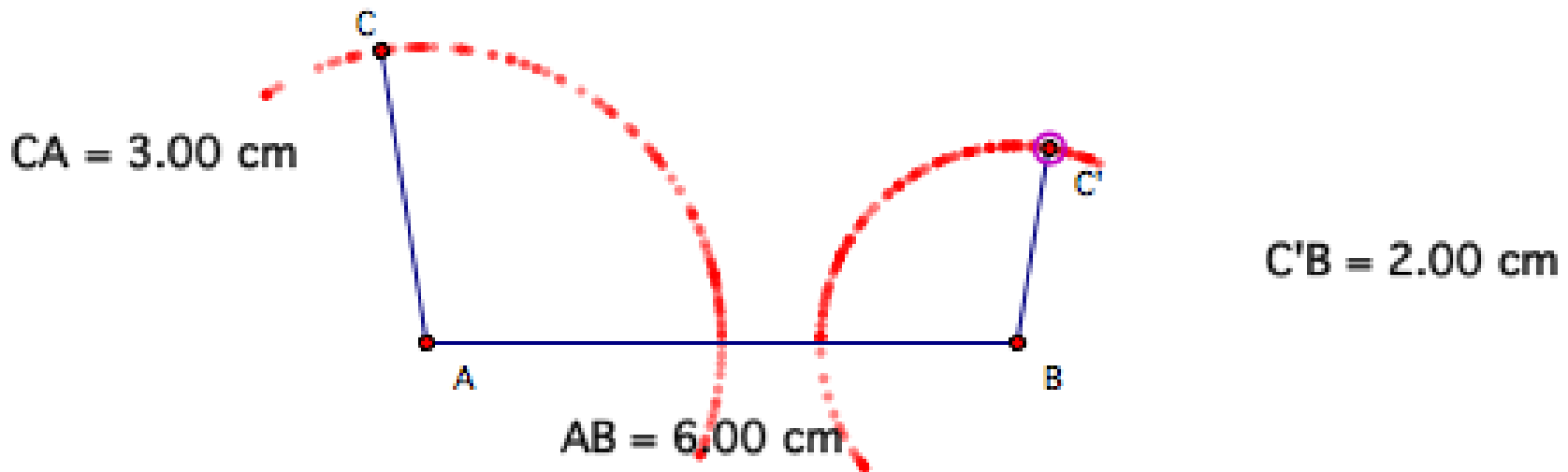
**3 x 3,**

**because of the  
Multiplication  
Counting  
Principle**

# Use appropriate tools strategically

## Grade 7

### Triangle Inequality Theorem



# Use appropriate tools strategically

## Teachers Should...

- Provide students with access to appropriate tools
- Facilitate students' selection of tools
- Help students become aware of the power of tools

# Process Standard#7

Look for and make use of structure

# Look for and make use of structure

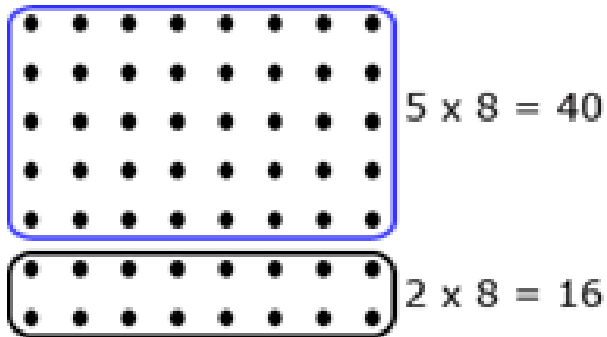
## Students should be able to...

- Explain mathematical patterns or structure
- Shift perspective and see things as single objects or as composed of several objects
- Explain why and when properties of operations are true in a context

# Look for and make use of structure

## 3<sup>rd</sup> grade

$$7 \times 8 = ?$$



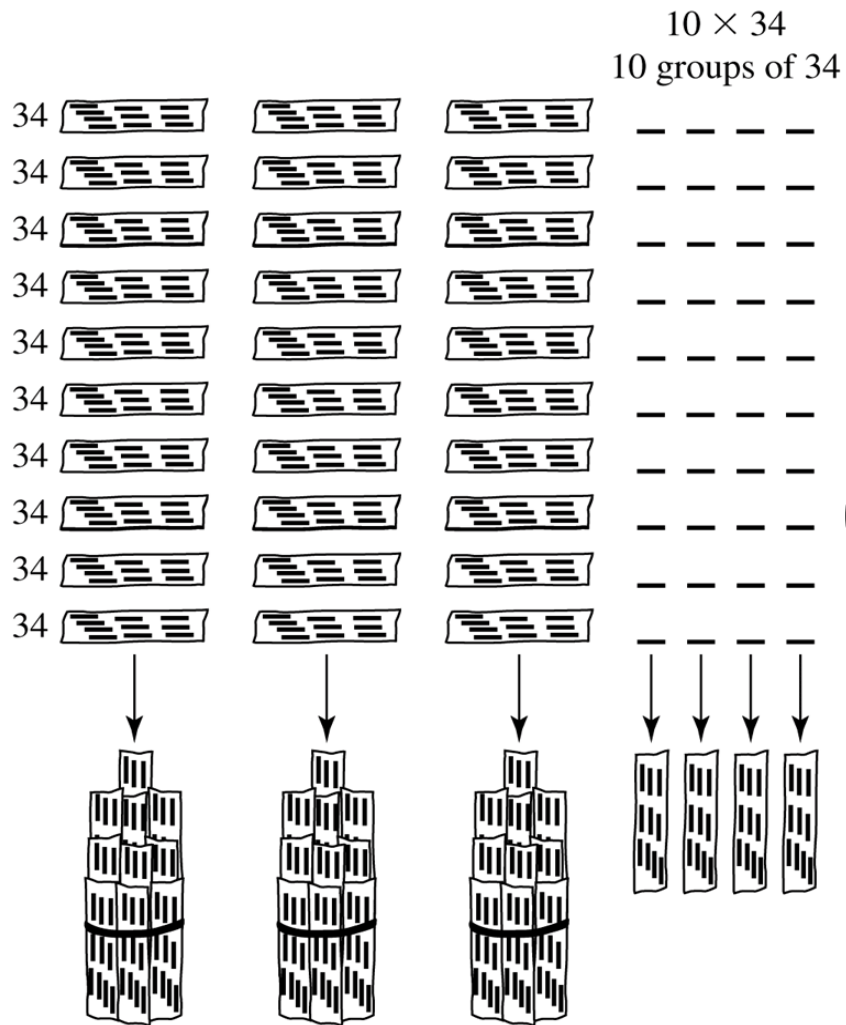
$$\text{Then } 40 + 16 = 56$$

I know that  
 $7 \times 7$  is 49. And  
one more 7  
makes 56.  
So  $8 \times 7 = 56$ .

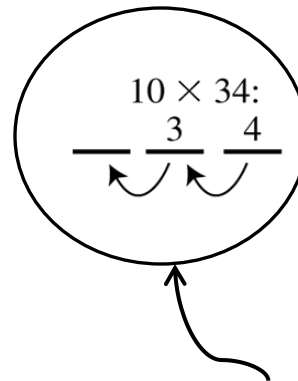


# Look for and make use of structure

## 5<sup>th</sup> grade



When we multiply a number by 10, the value of each digit becomes 10 times larger.



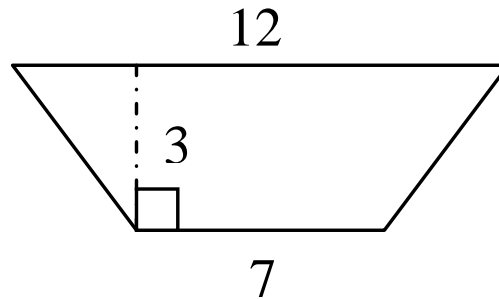
Each digit moves one place to the left.



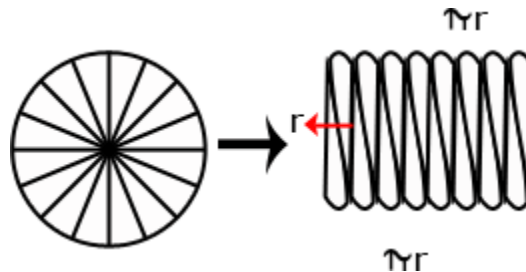
# Look for and make use of structure

## 7<sup>th</sup> grade

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Derive the formula for area of a circle using what you know about the area of a parallelogram.



# Look for and make use of structure

## Teachers should...

- Draw students' attention to the structure in mathematics.
  - ✓ Provide examples that are conducive for exploring structure
- Engage students in exploring patterns in numbers.
  - ✓ Patterns can be represented both visually and numerically and are a good context for recognizing structure
- Help students make use of structure.
  - ✓ Problems with a certain structure can be solved in a similar manner.

# Process Standard #8

Look for and express regularity in repeated reasoning

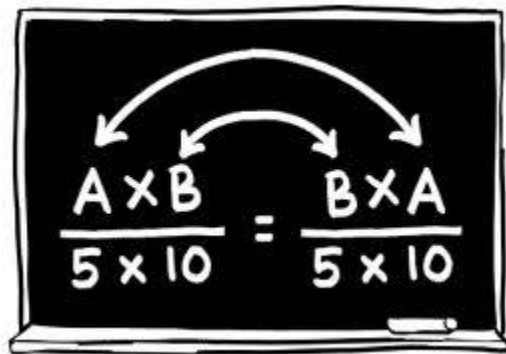
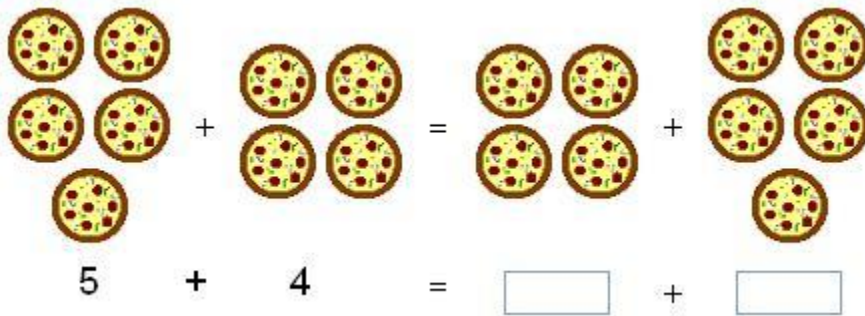


Look for and express regularity in repeated reasoning

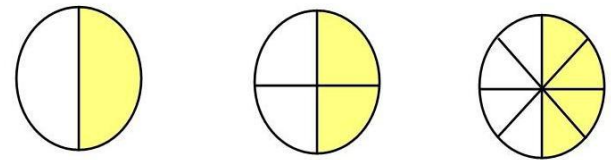
Students should be able to...

- Notice if calculations are repeated and use information to solve problems
- Use and justify the use of general methods or shortcuts
- Self-assess to see whether a strategy makes sense as they work, checking for reasonableness prior to getting the answer

# Regularity and repeated reasoning in mathematics is useful



$$\frac{7}{22} = 0.318181818181818 \dots$$



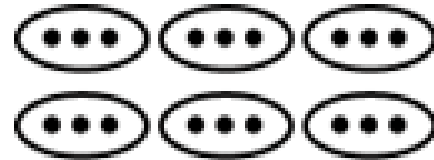
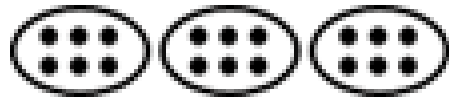
$\frac{1}{2} \xrightarrow{\times 2} \frac{2}{4} \xrightarrow{\times 2} \frac{4}{8}$

Look for and express regularity in repeated reasoning

3<sup>rd</sup> grade

- Commutative Property for Multiplication

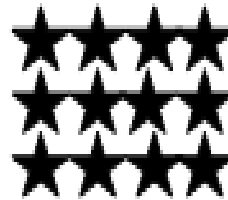
- For example:  $3 \times 6 = 6 \times 3$



- An array demonstrates the concept:



4 rows of 3 or  $4 \times 3$

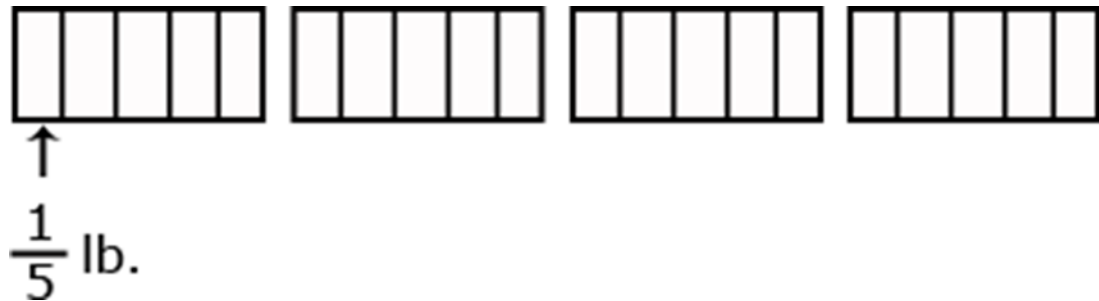


3 rows of 4 or  $3 \times 4$

Look for and express regularity in repeated reasoning

5<sup>th</sup> grade

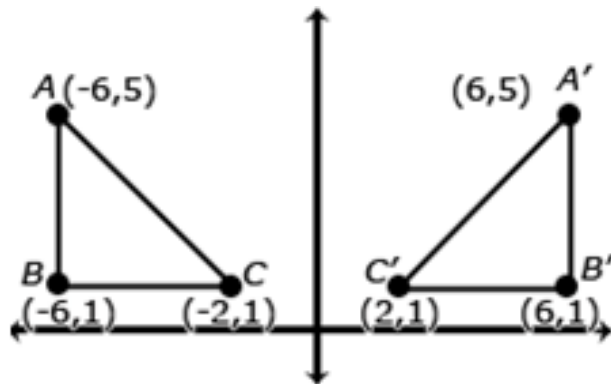
Angelo has 4 pounds of peanuts. He wants to give each of his friends  $\frac{1}{5}$  of a pound. How many friends can receive  $\frac{1}{5}$  of a pound of peanuts?



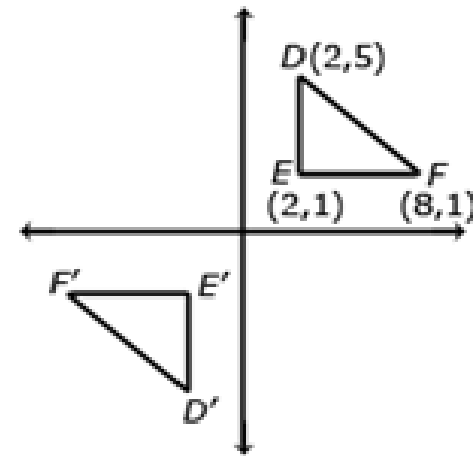
Look for and express regularity in repeated reasoning

8<sup>th</sup> grade

## Transformations



Reflection  
across the y-axis



180° Rotation  
about the Origin

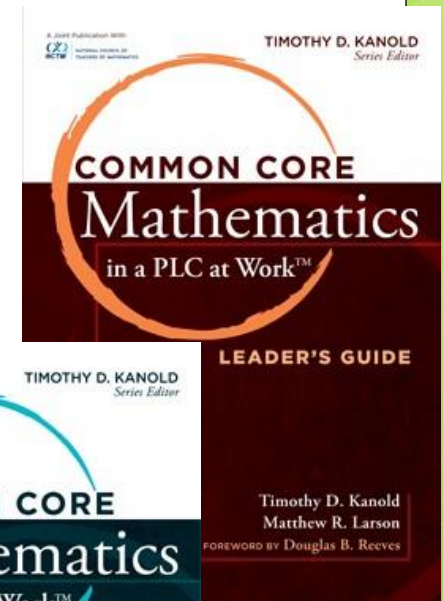
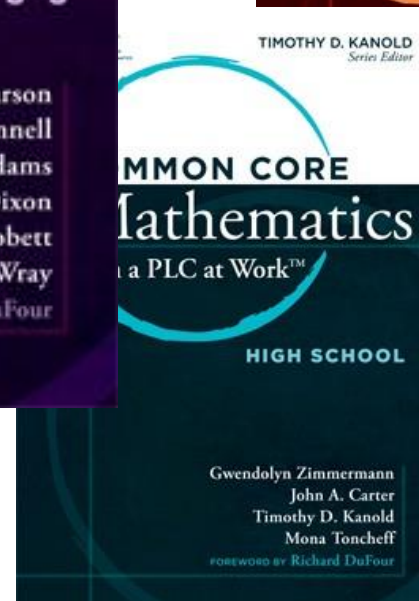
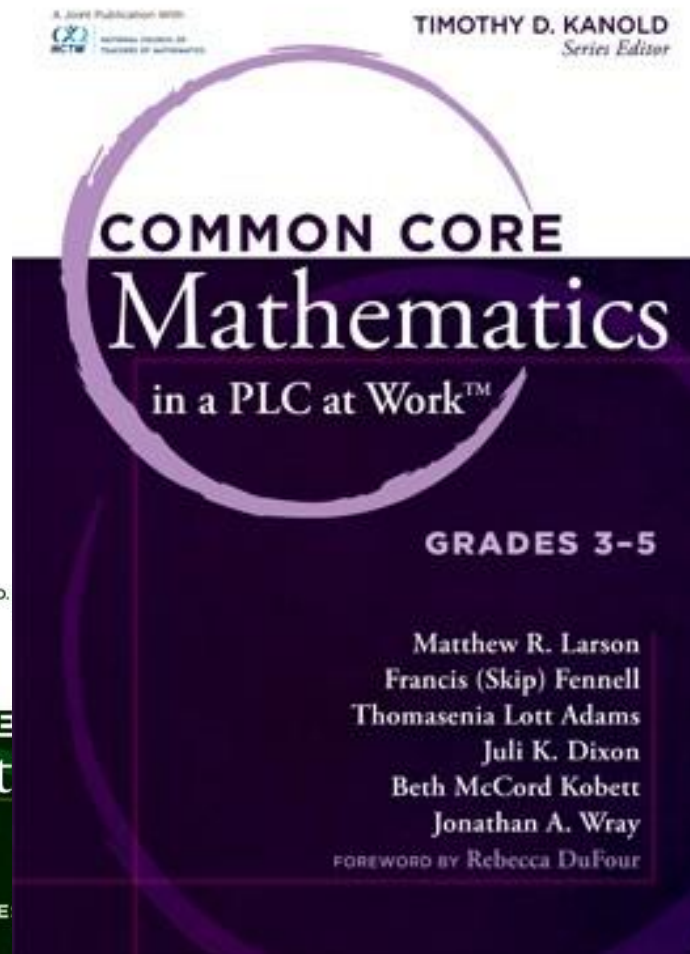
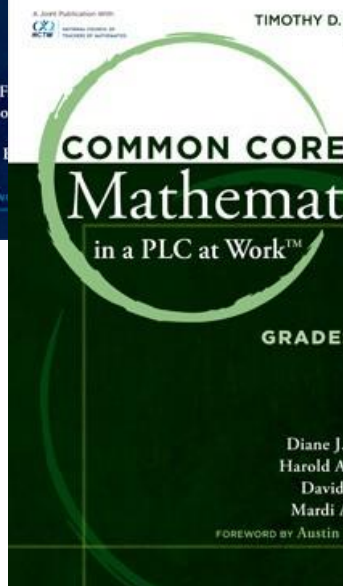
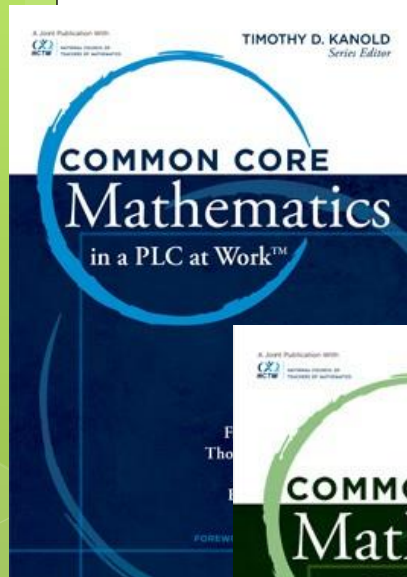


Look for and express regularity in repeated reasoning

## Teachers should...

- Avoid Teaching Shortcuts Before Students Develop Understanding of Important Concepts
  - ✓ Provide opportunities to look for the regularity in the calculations.
- Scaffold Examples to Highlight Regularity in Repeated Reasoning
  - ✓ Use multiple examples to help students move from seeing the repeated reasoning of a single example to being able to build a general method.
- Establish Expectations for Students, and Share Conjectures About General Methods
  - ✓ Establish norms in which students are expected to make and share conjectures related to what they notice

# Reference:





# Questions

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317-518-4577

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